Equation with sum of four sixth degree integers equal -

-to another four sixth degree integers

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Abstract

There are numerical solutions available on Wolfram world of mathematics website (ref. # 4) for the equation (p^6+q^6+r^6+s^6)=2(a^6+b^6). In this paper the author has arrived at numerical solution by algebra. It is common knowledge that arriving at numerical solutions by algebra is difficult for degree four & above. Also on the internet the author has not come across any method for the above mentioned equation.

Consider the below equation:

$$(p^6+q^6+r^6+s^6)=2(a^6+b^6)\ ---(1)$$

We have the Identity:

$$u^6+v^6=(x^6-3uvx^2(2x^2-3ab)-2(uv)^3)\ ----(2)$$

Where, x=(u+v)

In equation (1), we take, (a+b)=(p+q)=n & 5ab=3pq

Hence we have:

$$(a^6+a^6)=(n^6-3abn^2(2n^2-3ab)-2(ab)^3)---\ (3)$$

$$(p^6+q^6)=(n^6-3pqn^2(2n^2-3pq)-2(pq)^3)---\ (4)$$

Since, 3pq=5ab , eqn (4) becomes:

$$(p^6+q^6) = (n^6-5abn^2(2x^2-5ab)-2(pq)^3) --- (5)$$

From eqn (1) we have:

$$(r^6+s^6) = 2(a^6+b^6)-(p^6+q^6) ----- (7)$$

Substituting in the (RHS) of (7) from eqn (3) & (5) we get after some algebra:

$27(r^6+s^6) = 27n^6-54abn^4-189a^2b^2+142a^3b^3$ ----- (8)

$$= (3n^2-2ab)(9n^4-12abn^2-71a^2b^2) -- (9)$$

We now take, $x = 3r^2, \quad y = 3s^2$

Thus we have: x^3+y^3=$(3n^2-2ab)(9n^4-12abn^2-71a^2b^2)$

Hence we take,

$$(x+y) = (3n^2-2ab) ------------- (10)$$

$$(x^2+y^2) = (9n^4-12abn^2-46a^2b^2) ------(11)$$

Solving for (x,y) in in eqn (10) & (11) we notice that in-order to have integer

solution, the determinant "w" for [eqn (10) & (11)] is as below:

$$w^2 = (9n^4-12ab-96a^2b^2) ----- (11)$$

$$eqn\ (11)\ has\ numericalsolution, (a,b,n,w) = (9,2,11,273)$$

$$Hence, \qquad x = \frac{1}{2}(3n^2-2ab+w)$$

$$y = \frac{1}{2}(3n^2-2ab-w)$$

Substitutin for (a,b,n,w)=(9,2,11,273) in above we get: (x,y)=(300,27)

Since, $x = 3r^2, y = 3s^2 \quad we\ get: (r,s) = (10,3)$

& since, 3pq=5ab & (a,b)=(9,2)

We get (pxq)=30,

And since, $(p+q) = n = 11\ we\ get, \quad (p,q) = (6,5)$

$$Hence\ (a,b) = (9,2) \quad \& \quad (p,q,r,s) = (6,5,10,3)$$

Therefore: We have the below numerical solution:

$$(6^6 + 5^6 + 10^6 + 3^6) = 2(9^6 + 2^6)$$

References

1) Published math paper, author's Oliver Couto & Seiji Tomita, Generalized parametric solution to multiple sums of powers, Universal Journal of applied mathematics, Year July 2015, Volume 3(5),102-111, http://www.hrpub.org,

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2) Universal Journal of Applied Mathematics, Vol 4(3),45-65, Year OCT.2016, Parametric solutions to (six) nth powers equal to another (six) nth powers for n=(2,3,4,5,6,7,8,9). Author's Oliver Couto & Seiji Tomita

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3) Journal of number theory, #88, 225-240 (year 2001).

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3) Jaroslaw Wroblewski , Tables of Numerical solutions for various degrees,

Website, www.math.uni.wroc.pl/~jwr/eslp

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4) Wolfram mathworld: website: mathworld.wolfram.com
